CHAPTER 5  Stresses in Beams (Basic Topics)  

Problem 5.6-9  A beam ABC with an overhang from B to C is constructed of a C 10 × 30 channel section (see figure). The beam supports its own weight (30 lb/ft) plus a triangular load of maximum intensity \( q_0 \) acting on the overhang. The allowable stresses in tension and compression are 20 ksi and 11 ksi, respectively.

Determine the allowable triangular load intensity \( q_{0,\text{allow}} \) if the distance \( L \) equals 3.5 ft.

\[
\begin{align*}
\text{Solution 5.6-9} \\
\text{Numerical data} \\
w = 30 \frac{\text{lb}}{\text{ft}} & \quad \sigma_{at} = 20 \text{ ksi} & \quad \sigma_{ac} = 11 \text{ ksi} \\
L = 3.5 \text{ ft} & \\
c_1 = 2.384 \text{ in.} & \quad c_2 = 0.649 \text{ in.} \\
\text{from Table E-3(a)} & \quad I_{zz} = 3.93 \text{ in.}^4 \\
\text{Max. moment is at B (tension top, compression bottom).} & \\
M_B = wL \frac{L}{2} + \frac{1}{2} q_0L \left( \frac{2}{3} L \right) & \\
M_B = \frac{1}{2} wL^2 + \frac{1}{3} q_0L^2 \\
\text{Check tension on top} & \\
\sigma_t = \frac{M_B c_1}{I_{zz}} & \quad M_B = \sigma_{at} \frac{I_{zz}}{c_1} \\
q_{allow} = \frac{3}{L} \left[ \sigma_{at} \frac{I_{zz}}{c_1} - \frac{1}{2} wL^2 \right] & \quad \text{governs} \\
q_{allow} = 628 \text{ lb/ft} & \\
\text{Check compression on bottom} & \\
q_{allow} = \frac{3}{L} \left[ \sigma_{ac} \frac{I_{zz}}{c_1} - \frac{1}{2} wL^2 \right] & \quad \text{governs} \\
q_{allow} = 1314 \frac{\text{lb}}{\text{ft}}
\end{align*}
\]

Problem 5.6-10  A so-called “trapeze bar” in a hospital room provides a means for patients to exercise while in bed (see figure). The bar is 2.1 m long and has a cross section in the shape of a regular octagon. The design load is 1.2 kN applied at the midpoint of the bar, and the allowable bending stress is 200 MPa.

Determine the minimum height \( h \) of the bar. (Assume that the ends of the bar are simply supported and that the weight of the bar is negligible.)

\[
\begin{align*}
\text{Diagram:} \\
C & \quad h \\
\end{align*}
\]
Problem 5.6-11  A two-axle carriage that is part of an overhead traveling crane in a testing laboratory moves slowly across a simple beam AB (see figure). The load transmitted to the beam from the front axle is 2200 lb and from the rear axle is 3800 lb. The weight of the beam itself may be disregarded.

(a) Determine the minimum required section modulus S for the beam if the allowable bending stress is 17.0 ksi, the length of the beam is 18 ft, and the wheelbase of the carriage is 5 ft.

(b) Select the most economical I-beam (S shape) from Table E-2(a), Appendix E.

Solution 5.6-11

Numerical data

\[ L = 18 \text{ ft} \quad P_1 = 2200 \text{ lb} \]

\[ P_2 = 3800 \text{ lb} \quad d = 5 \text{ ft} \]

\[ \sigma_d = 17 \text{ ksi} \]

(a) Find reaction \( R_A \) then an expression for moment under larger load \( P_2 \); let \( x = \text{dist. from } A \) to load \( P_2 \)

\[ R_A = P_2 \left( \frac{L - x}{L} \right) + P_1 \left[ \frac{L - (x + d)}{L} \right] \]

\[ M_2 = R_A x \]

\[ M_2 = x \left[ P_2 \left( \frac{L - x}{L} \right) + P_1 \left[ \frac{L - (x + d)}{L} \right] \right] \]

\[ M_2 = \frac{xP_2L - P_2x^2 + xP_1L - P_1x^2 - xP_1d}{L} \]

Take derivative of \( M_A \) & set to zero to find max. bending moment at \( x = x_m \)

\[ \frac{d}{dx} \left( \frac{xP_2L - P_2x^2 + xP_1L - P_1x^2 - xP_1d}{L} \right) = \frac{P_2L - 2P_2x + P_1L - 2P_1x - P_1d}{L} \]

\[ P_2L - 2P_2x + P_1L - 2P_1x - P_1d = 0 \]

\[ x_m = \frac{(P_1 + P_2)L - P_1d}{2(P_1 + P_2)} \quad x_m = 8.083 \text{ ft} \]

\[ R_A = P_2 \left( \frac{L - x_m}{L} \right) + P_1 \left[ \frac{L - (x_m + d)}{L} \right] \]

\[ R_A = 2694 \text{ lb} \]

\[ M_{\text{max}} = x_m \left[ P_2 \left( \frac{L - x_m}{L} \right) + P_1 \left[ \frac{L - (x_m + d)}{L} \right] \right] \]

\[ M_{\text{max}} = 21780 \text{ ft-lb} \]

\[ S_{\text{reqd}} = \frac{M_{\text{max}}}{\sigma_d} \quad S_{\text{reqd}} = 15.37 \text{ in.}^3 \]

(b) Select most economical S shape from Table E-2(a)

select S8 × 23 \[ S_{\text{act}} = 16.2 \text{ in.}^3 \]

Problem 5.6-12  A cantilever beam AB of circular cross section and length \( L = 450 \text{ mm} \) supports a load \( P = 400 \text{ N} \) acting at the free end (see figure). The beam is made of steel with an allowable bending stress of 60 MPa.

Determine the required diameter \( d_{\text{min}} \) of the beam, considering the effect of the beam's own weight.
Solution 5.6-12  Cantilever beam

**DATA**

- \( L = 450 \text{ mm} \)
- \( P = 400 \text{ N} \)
- \( \sigma_{\text{allow}} = 60 \text{ MPa} \)
- \( \gamma = \text{weight density of steel} = 77.0 \text{ kN/m}^3 \)

**Weight of beam per unit length**

\[
q = \gamma \left( \frac{\pi d^2}{4} \right)
\]

**Maximum bending moment**

\[
M_{\text{max}} = PL + \frac{q L^2}{2} = PL + \frac{\pi \gamma d^2 L^2}{8}
\]

**Section modulus**

\[
S = \frac{\pi d^3}{32}
\]

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**Problem 5.6-13**

A compound beam ABCD (see figure) is supported at points A, B, and D and has a splice at point C. The distance \( a = 6.25 \text{ ft} \), and the beam is a \( S 18 \times 70 \) wide-flange shape with an allowable bending stress of 12,800 psi.

(a) If the splice is a moment release, find the allowable uniform load \( q_{\text{allow}} \) that may be placed on top of the beam, taking into account the weight of the beam itself.

[See figure part (a).]

(b) Repeat assuming now that the splice is a shear release, as in figure part (b).

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**Solution 5.6-13**

**Numerical data**

- \( w = 70 \text{ lb/ft} \)
- \( S = 103 \text{ in.}^3 \)
- \( a = 6.25 \text{ ft} \)
- \( \sigma_a = 12800 \text{ psi} \)

(a) **Moment release at C—gives max. moment at B**

(see moment diagram) \( = -2.5 q a^2 \)

\[
\sigma_a = \frac{M_{\text{max}}}{S} \quad M_{\text{max}} = [(q_{\text{allow}} + w) a^2 (2.5)]
\]

and \( M_{\text{max}} = \sigma_a S \)

\[
w = 70 \text{ lb/ft} \quad S = 103 \text{ in.}^3 \]

\[
a = 6.25 \text{ ft} \quad \sigma_a = 12800 \text{ psi}
\]
Solution 5.8-6  Beam of rectangular cross section

\( b = \text{width} \quad h = \text{height} \quad L = \text{length} \)

Uniform load \( q = \text{intensity of load} \)

**ALLOWABLE STRESSES** \( \sigma_{\text{allow}} \) and \( \tau_{\text{allow}} \)

(a) **SIMPLE BEAM**

**BENDING**

\[ M_{\text{max}} = \frac{qL^2}{2} \quad S = \frac{bh^2}{6} \]

\[ \sigma_{\text{max}} = \frac{M_{\text{max}}}{S} = \frac{3qL^2}{4bh^2} \]

\[ q_{\text{allow}} = \frac{\sigma_{\text{allow}}bh^2}{3L^2} \]  \hspace{1cm} (1)

**SHEAR**

\[ V_{\text{max}} = \frac{qL}{2} \quad A = bh \]

\[ \tau_{\text{max}} = \frac{3V}{2A} = \frac{3qL}{2bh} \]

\[ q_{\text{allow}} = \frac{4\tau_{\text{allow}}bh}{3L} \]  \hspace{1cm} (2)

Equate (1) and (2) and solve for \( L_0 \):

\[ L_0 = h\left(\frac{\sigma_{\text{allow}}}{\tau_{\text{allow}}}\right) \quad \leftarrow \]

(b) **CANTILEVER BEAM**

**BENDING**

\[ M_{\text{max}} = \frac{qL^2}{2} \quad S = \frac{bh^2}{6} \]

\[ \sigma_{\text{max}} = \frac{M_{\text{max}}}{S} = \frac{3qL^2}{bh^2} \]

\[ q_{\text{allow}} = \frac{\sigma_{\text{allow}}bh^2}{3L^2} \]  \hspace{1cm} (3)

**SHEAR**

\[ V_{\text{max}} = qL \quad A = bh \]

\[ \tau_{\text{max}} = \frac{3V}{2A} = \frac{3qL}{2bh} \]

\[ q_{\text{allow}} = \frac{2\tau_{\text{allow}}bh}{3L} \]  \hspace{1cm} (4)

Equate (3) and (4) and solve for \( L_0 \):

\[ L_0 = h\left(\frac{\sigma_{\text{allow}}}{\tau_{\text{allow}}}\right) \quad \leftarrow \]

**NOTE:** If the actual length is less than \( L_0 \), the shear stress governs the design. If the length is greater than \( L_0 \), the bending stress governs.

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**Problem 5.8-7** A laminated wood beam on simple supports is built up by gluing together four 2 in. \( \times \) 4 in. boards (actual dimensions) to form a solid beam 4 in. \( \times \) 8 in. in cross section, as shown in the figure. The allowable shear stress in the glued joints is 65 psi, and the allowable bending stress in the wood is 1800 psi.

If the beam is 9 ft long, what is the allowable load \( P \) acting at the one-third point along the beam as shown? (Include the effects of the beam's own weight, assuming that the wood weighs 35 lb/ft\(^3\).)

**Solution 5.8-7**

\( L = 9 \text{ ft} \quad b = 4 \text{ in.} \)

\( h = 8 \text{ in.} \quad A = bh \)

\( \tau_{\text{allow}} = 65 \text{ psi} \quad \sigma_{\text{allow}} = 1800 \text{ psi} \)

**WEIGHT OF BEAM PER UNIT DISTANCE**

\( \gamma = 35 \text{ lb/ft}^3 \quad q = \gamma A \)

\( q = 7.778 \frac{1\text{ lb}}{\text{ft}} \)

**ALLOWABLE LOAD BASED UPON SHEAR STRESS IN THE GLUED JOINTS; MAX. SHEAR STRESS AT NEUTRAL AXIS**

\( \tau = \frac{VQ}{lb} \quad \tau_{\text{max}} = \frac{3V}{2A} \)
\[ V = P \frac{2}{3} + \frac{qL}{2} \]

\[ \tau_{\text{max}} = \frac{3V}{2A} = \frac{3}{2A} \left( P \frac{2}{3} + \frac{qL}{2} \right) \]

\[ P = \left( A \tau_{\text{max}} - \frac{3qL}{4} \right) \]

\[ P_{\text{max}} = A \tau_{\text{allow}} - \frac{3qL}{4} \]

\[ P_{\text{max}} = 2.03 \, k \, (\text{governs}) \]

**Problem 5.8-8** A laminated plastic beam of square cross section is built up by gluing together three strips, each 10 mm \( \times \) 30 mm in cross section (see figure). The beam has a total weight of 3.6 N and is simply supported with span length \( L = 360 \) mm.

Considering the weight of the beam \( (q) \) calculate the maximum permissible CCW moment \( M \) that may be placed at the right support.

(a) If the allowable shear stress in the glued joints is 0.3 MPa.

(b) If the allowable bending stress in the plastic is 8 MPa.

**Solution 5.8-8**

(a) **Find \( M \) based on allowable shear stress in glued joint**

\[ b = 30 \text{ mm} \quad h = 30 \text{ mm} \quad \tau_a = 0.3 \, \text{MPa} \]

\[ W = 3.6 \, \text{N} \quad L = 360 \, \text{mm} \]

\[ q = \frac{W}{L} \]

\[ q = 10 \, \frac{\text{N}}{\text{m}} \quad \text{beam distributed weight} \]

**Max. shear at left support**

\[ V_m = \frac{qL}{2} + \frac{M}{L} \quad \text{and} \quad V_m = \tau_a \left( \frac{lb}{Q} \right) \]

\[ \tau_a = \frac{V_m Q}{lb} \quad l = \frac{bh^3}{12} \quad lb = \frac{bh^3}{12} \]

**Allowable load based upon bending stress**

\[ M = P \frac{2}{3} \text{ ft} + \frac{qL}{2} \text{ ft} - \frac{q}{2} \left( \text{ft} \right)^2 \]

\[ S = \frac{bh^2}{6} \]

\[ \sigma_{\text{max}} = \frac{M}{S} = \frac{P \frac{2}{3} \text{ ft} + \frac{qL}{2} \text{ ft} - \frac{q}{2} \left( \text{ft} \right)^2}{S} \]

\[ P_{\text{max}} = \sigma_{\text{allow}} S^3 = \frac{3}{2} \left( \frac{qL}{2} - \frac{q}{2} \left( \text{ft} \right) \right) \]

\[ P_{\text{max}} = 3.165 \, k \]

\[ P_{\text{allow}} = 2.03 \, k \]

(b) **Find \( M \) based on allowable bending stress**

\[ Q = \frac{bh}{3} \quad Q = \frac{bh^2}{9} \quad \frac{Q}{lb} = \frac{bh^2}{9} \]

\[ \frac{Q}{lb} = \frac{4}{3bh} \]

\[ M = L \left[ \tau_a \left( \frac{lb}{Q} \right) - \frac{qL}{2} \right] \]

\[ M = L \left[ \tau_a \left( \frac{3bh}{4} \right) - \frac{qL}{2} \right] \]

\[ M_{\text{max}} = 72.2 \, \text{N} \cdot \text{M} \]
Maximum moment based upon wood
\[ \sigma_{allow, w} = 7 \text{ MPa} \]
From \[ \sigma_{allow, w} = \frac{M_{allow, w}(h - h_1)E_w}{E_wI_1 + E_sI_2} \]
\[ M_{allow, w} = 18.68 \text{ kN} \cdot \text{m} \]

Maximum moment based upon steel
\[ \sigma_{allow, s} = 120 \text{ MPa} \]
From \[ \sigma_{allow, s} = \frac{M_{allow, s}(h - h_1)E_s}{E_uI_1 + E_sI_2} \]
\[ M_{allow, s} = 16.78 \text{ kN} \cdot \text{m} \]

Maximum allowable moment
\[ M_{allow} = \min(M_{allow, w}, M_{allow, s}) \]
Steel governs. \[ M_{allow} = 16.78 \text{ kN} \cdot \text{m} \]

Maximum uniform distributed load
From \[ M_{allow} = \frac{q_{max}L^2}{12} \]
\[ q_{max} = 15.53 \text{ kN/m} \]

Transformed-Section Method

When solving the problems for Section 6.3, assume that the component parts of the beams are securely bonded by adhesives or connected by fasteners. Also, be sure to use the transformed-section method in the solutions.

Problem 6.3-1 A wood beam 8 in. wide and 12 in. deep (nominal dimensions) is reinforced on top and bottom by 0.25-in.-thick steel plates (see figure part a).

(a) Find the allowable bending moment \( M_{max} \) about the \( z \) axis if the allowable stress in the wood is 1,100 psi and in the steel is 15,000 psi. (Assume that the ratio of the moduli of elasticity of steel and wood is 20.)

(b) Compare the moment capacity of the beam in part a with that shown in the figure part b which has two 4 in. \( \times \) 12 in. joists (nominal dimensions) attached to a 1/4 in. \( \times \) 11.0 in. steel plate.
Solution 6.3-1
(a) FIND $M_{\text{max}}$

(1) Wood beam $b = 7.5$ in. $h_1 = 11.5$ in.
\[ \sigma_{\text{allow, w}} = 1100 \text{ psi} \]

(2) Steel plates $b = 7.5$ in. $h_2 = 12$ in.
\[ t = 0.25 \text{ in.} \]
\[ \sigma_{\text{allow, s}} = 15000 \text{ psi} \]

**TRANSFORMED SECTION (WOOD)**

\[ n = 20 \]

**WIDTH OF STEEL PLATES**

\[ b_T = nt \quad b_T = 5 \text{ in.} \]
\[ I_T = \frac{bh_T^3}{12} + \frac{t b_T (h_2 - t)^2}{2} \]
\[ I_T = 3540 \text{ in.}^4 \]

**MAXIMUM MOMENT BASED UPON THE WOOD (1)**

\[ M_1 = \frac{\sigma_{\text{allow, w}} I_T}{h_1} \quad M_1 = 677 \text{ k \cdot in.} \]

**MAXIMUM MOMENT BASED UPON THE STEEL (2)**

\[ M_2 = \frac{\sigma_{\text{allow, s}} I_T}{h_2 t} \quad M_2 = 442 \text{ k \cdot in.} \]

\[ M_{\text{max}} = \min(M_1, M_2) \]

**STELL GOVERNS**

\[ M_{\text{max}} = 422 \text{ k-in.} \]

(b) **COMPARE MOMENT CAPACITIES**

(1) Wood beam $b = 3.5$ in. $h_1 = 11.25$ in.

(2) Steel plates $h_2 = 11$ in. $t = 0.25$ in.

**WIDTH OF STEEL PLATES**

\[ b_T = nt \quad b_T = 5 \text{ in.} \]
\[ I_T = \frac{bh_T^3}{12} + \frac{t b_T h_T^2}{12} \quad I_T = 1385 \text{ in.}^4 \]

**MAXIMUM MOMENT BASED UPON THE WOOD (1)**

\[ M_1 = \frac{\sigma_{\text{allow, w}} I_T}{h_1} \quad M_1 = 271 \text{ k \cdot in.} \]

**MAXIMUM MOMENT BASED UPON THE STEEL (2)**

\[ M_2 = \frac{\sigma_{\text{allow, s}} I_T}{h_2 t} \quad M_2 = 189 \text{ k \cdot in.} \]

\[ M_{\text{max}} = \min(M_1, M_2) \]

**STELL GOVERNS**

\[ M_{\text{max}} = 189 \text{ k-in.} \]

The moment capacity of the beam in (a) is 2.3 times more than the beam in (b).

**Problem 6.3-2** A simple beam of span length 3.2 m carries a uniform load of intensity 48 kN/m. The cross section of the beam is a hollow box with wood flanges and steel side plates, as shown in the figure. The wood flanges are 75 mm by 100 mm in cross section, and the steel plates are 300 mm deep.

What is the required thickness $t$ of the steel plates if the allowable stresses are 120 MPa for the steel and 6.5 MPa for the wood? (Assume that the moduli of elasticity for the steel and wood are 210 GPa and 10 GPa, respectively, and disregard the weight of the beam.)