Problem 1.2-3 A bicycle rider would like to compare the effectiveness of cantilever hand brakes [see figure part (a)] versus V brakes [figure part (b)].

(a) Calculate the braking force $R_B$ at the wheel rims for each of the bicycle brake systems shown. Assume that all forces act in the plane of the figure and that cable tension $T = 45$ lbs. Also, what is the average compressive normal stress $\sigma_c$ on the brake pad ($A = 0.625 \text{ in.}^2$)?

(b) For each braking system, what is the stress in the brake cable (assume effective cross-sectional area of 0.00167 in.$^2$)? (HINT: Because of symmetry, you only need to use the right half of each figure in your analysis.)

Solution 1.2-3

$T = 45 \text{ lbs} \quad A_{pad} = 0.625 \text{ in.}^2$

$A_{cable} = 0.00167 \text{ in.}^2$

(a) CANTILEVER BRAKES-BRAKING FORCE

$R_B$ & PAD PRESSURE

Statics: sum forces at D to get $T_{DC} = T/2$

$\sum M_A = 0$

$R_B(1) = T_{DCM}(3) + T_{DCV}(1)$

$T_{DCM} = T_{DCV} \quad T_{DCm} = T/2$

$R_B = 2T \quad R_B = 90 \text{ lbs}$

so $R_B = 2T$ vs. $4.25T$ for V brakes (below)
CHAPTER 1  Tension, Compression, and Shear

\[ \sigma_{pad} = \frac{R_B}{A_{pad}} \quad \sigma_{pad} = 144 \text{ psi} \]
\[ \sigma_{cable} = \frac{T}{A_{cable}} \quad \sigma_{cable} = 26,946 \text{ psi} \]
\[ \frac{4.25}{2} = 2.125 \]
\[ \text{(same for V brakes (below))} \]

(b) \text{ V brakes - braking force } R_B \& \text{ pad pressure}

\[ \sum M_A = 0 \quad R_B = 4.25T \quad R_B = 191.3 \text{ lbs} \]

\[ \sigma_{pad} = \frac{R_B}{A_{pad}} \quad \sigma_{pad} = 306 \text{ psi} \]

Problem 1.2-4  A circular aluminum tube of length
\[ L = 400 \text{ mm} \] is loaded in compression by forces \( P \) (see figure). The outside and inside diameters are 60 mm and 50 mm, respectively. A strain gage is placed on the outside of the bar to measure normal strains in the longitudinal direction.

(a) If the measured strain in \( \varepsilon = 550 \times 10^{-6} \), what is the shortening \( \delta \) of the bar?

(b) If the compressive stress in the bar is intended to be 40 MPa, what should be the load \( P \)?

Solution 1.2-4  Aluminum tube in compression

\[ \varepsilon = 550 \times 10^{-6} \]
\[ L = 400 \text{ mm} \]
\[ d_2 = 60 \text{ mm} \]
\[ d_1 = 50 \text{ mm} \]

(a) Shortening \( \delta \) of the bar
\[ \delta = \varepsilon L = (550 \times 10^{-6}) \times (400 \text{ mm}) \]
\[ = 0.220 \text{ mm} \]

(b) Compressive load \( P \)
\[ \sigma = 40 \text{ MPa} \]
\[ A = \frac{\pi}{4} [d_2^2 - d_1^2] = \frac{\pi}{4} [(60 \text{ mm})^2 - (50 \text{ mm})^2] \]
\[ P = \sigma A = (40 \text{ MPa})(863.9 \text{ mm}^2) \]
\[ = 34.6 \text{ kN} \]
Solution 1.5-4

NUMERICAL DATA

\[ P = 65 \text{ kN} \quad \nu = \frac{1}{3} \]

\[ d = 32 \text{ mm} \quad L = 1.75(1000) \text{ mm} \]

\[ E = 75 \text{ GPa} \]

INITIAL AREA OF CROSS SECTION

\[ A_i = \frac{\pi d^2}{4} \quad A_i = 804.248 \text{ mm}^2 \]

AXIAL STRAIN

\[ \varepsilon = \frac{P}{E A_i} \quad \varepsilon = 1.078 \times 10^{-3} \]

INCREASE IN LENGTH

\[ \Delta L = \varepsilon L \quad \Delta L = 1.886 \text{ mm} \]

LATERNAL STRAIN

\[ \varepsilon_p = -\nu \varepsilon \quad \varepsilon_p = -3.592 \times 10^{-4} \]

DECREASE IN DIAMETER

\[ \Delta d = |\varepsilon_p| \quad \Delta d = 0.011 \text{ mm} \]

FINAL AREA OF CROSS SECTION

\[ A_f = \frac{\pi}{4} (d - \Delta d)^2 \]

\[ A_f = 803.67 \text{ mm}^2 \]

\[ \% \text{ decrease in x-sec area} = \frac{A_f - A_i}{A_i} (100) \rightarrow \]

\[ = -0.072 \rightarrow \]

Problem 1.5-5 A bar of monel metal as in the figure (length \( L = 9 \text{ in.} \), diameter \( d = 0.225 \text{ in.} \)) is loaded axially by a tensile force \( P \). If the bar elongates by 0.0195 in., what is the decrease in diameter \( d \)? What is the magnitude of the load \( P \)? Use the data in Table H-2, Appendix H.

Solution 1.5-5

NUMERICAL DATA

\[ E = 25000 \text{ ksi} \]

\[ \nu = 0.32 \]

\[ L = 9 \text{ in.} \]

\[ \delta = 0.0195 \text{ in.} \]

\[ d = 0.225 \text{ in.} \]

NORMAL STRAIN

\[ \varepsilon = \frac{\delta}{L} \quad \varepsilon = 2.167 \times 10^{-3} \]

LATERNAL STRAIN

\[ \varepsilon_p = -\nu \varepsilon \quad \varepsilon_p = -6.933 \times 10^{-4} \]

DECREASE IN DIAMETER

\[ \Delta d = \varepsilon_p d \]

\[ \Delta d = -1.56 \times 10^{-4} \text{ in.} \rightarrow \]

INITIAL CROSS SECTIONAL AREA

\[ A_i = \frac{\pi}{4} d^2 \quad A_i = 0.04 \text{ in.}^2 \]

MAGNITUDE OF LOAD \( P \)

\[ P = EA_i \varepsilon \]

\[ P = 2.15 \text{ kips} \rightarrow \]
Problem 2.4-2 A cylindrical assembly consisting of a brass core and an aluminum collar is compressed by a load $P$ (see figure). The length of the aluminum collar and brass core is 350 mm, the diameter of the core is 25 mm, and the outside diameter of the collar is 40 mm. Also, the moduli of elasticity of the aluminum and brass are 72 GPa and 100 GPa, respectively.

(a) If the length of the assembly decreases by 0.1% when the load $P$ is applied, what is the magnitude of the load?
(b) What is the maximum permissible load $P_{\text{max}}$ if the allowable stresses in the aluminum and brass are 80 MPa and 120 MPa, respectively? (Suggestion: Use the equations derived in Example 2-5.)

Solution 2.4-2 Cylindrical assembly in compression

\[ \Delta = \frac{PL}{E_aA_a + E_bA_b} \quad \text{or} \quad P = (E_aA_a + E_bA_b)\left(\frac{\Delta}{L}\right) \]

Substitute numerical values:

\[
E_aA_a + E_bA_b = (72 \text{ GPa})(765.8 \text{ mm}^2) \\
+ (100 \text{ GPa})(490.9 \text{ mm}^2) \\
= 55,135 \text{ MN} + 49,090 \text{ MN} \\
= 104.23 \text{ MN}
\]

\[
P = (104.23 \text{ MN})\left(\frac{0.350 \text{ mm}}{350 \text{ mm}^2}\right) \\
= 104.2 \text{ kN} \quad \leftarrow (b) \text{ Allowable load}
\]

\[
\sigma_a = \frac{80 \text{ MPa}}{72 \text{ GPa}} \\
\sigma_b = \frac{120 \text{ MPa}}{100 \text{ GPa}}
\]

Use Eqs. (2-12a and b) of Example 2-5.

For aluminum:

\[
P_a = \frac{PE_a}{E_aA_a + E_bA_b} \\
P_a = (E_aA_a + E_bA_b)\left(\frac{\sigma_a}{E_a}\right) \\
P_a = (104.23 \text{ MN})\left(\frac{80 \text{ MPa}}{72 \text{ GPa}}\right) = 115.8 \text{ kN}
\]

For brass:

\[
P_b = \frac{PE_b}{E_aA_a + E_bA_b} \\
P_b = (E_aA_a + E_bA_b)\left(\frac{\sigma_b}{E_b}\right) \\
P_b = (104.23 \text{ MN})\left(\frac{120 \text{ MPa}}{100 \text{ GPa}}\right) = 125.1 \text{ kN}
\]

Aluminum governs. $P_{\text{max}} = 116 \text{ kN} \quad \leftarrow$
Problem 2.5-20  A plastic cylinder is held snugly between a rigid plate and a foundation by two steel bolts (see figure).

Determine the compressive stress $\sigma_p$ in the plastic when the nuts on the steel bolts are tightened by one complete turn.

Data for the assembly are as follows: length $L = 200$ mm, pitch of the bolt threads $p = 1.0$ mm, modulus of elasticity for steel $E_s = 200$ GPa, modulus of elasticity for the plastic $E_p = 7.5$ GPa, cross-sectional area of one bolt $A_s = 36.0$ mm$^2$, and cross-sectional area of the plastic cylinder $A_p = 960$ mm$^2$.

Solution 2.5-20  Plastic cylinder and two steel bolts

$L = 200$ mm
$P = 1.0$ mm
$E_s = 200$ GPa
$A_s = 36.0$ mm$^2$ (for one bolt)
$E_p = 7.5$ GPa
$A_p = 960$ mm$^2$

$\gamma = 1$ (See Eq. 2.22)

Equilibrium equation

$P_s - P_s + P_p = 0$

$P_s = \text{tensile force in one steel bolt}$
$P_p = \text{compressive force in plastic cylinder}$
$P_p = 2P_s$  \hspace{1cm} \text{(Eq. 1)}

Compatibility equation

$\delta_s = \text{elongation of steel bolt}$
$\delta_p = \text{shortening of plastic cylinder}$
$\delta_s + \delta_p = np$  \hspace{1cm} \text{(Eq. 2)}

Force-displacement relations

$\delta_s = \frac{P_s L}{E_s A_s}$
$\delta_p = \frac{P_p L}{E_p A_p}$  \hspace{1cm} \text{(Eq. 3, Eq. 4)}

Solutions of equations

Substitute (3) and (4) into Eq. (2):

$\frac{P_s L}{E_s A_s} + \frac{P_p L}{E_p A_p} = np$  \hspace{1cm} \text{(Eq. 5)}

Solve simultaneously Eqs. (1) and (5):

$P_p = \frac{2npE_s A_s E_p A_p}{L(E_p A_p + 2E_s A_s)}$
STRESS IN THE PLASTIC CYLINDER

\[ \sigma_p = \frac{P_p}{A_p} = \frac{2np}{L(E_p A_p + 2E_s A_s)} \]

SUBSTITUTE NUMERICAL VALUES:

\[ N = E_s A_s E_p = 54.0 \times 10^{15} \text{ N/mm}^2 \]

\[ D = E_p A_p + 2E_s A_s = 21.6 \times 10^6 \text{ N} \]

\[ \sigma_p = \frac{2np}{L} \left( \frac{N}{D} \right) = \frac{2(1)(1.0 \text{ mm})}{200 \text{ mm}} \left( \frac{N}{D} \right) = 25.0 \text{ MPa} \]

Problem 2.5-21  Solve the preceding problem if the data for the assembly are as follows: length \( L = 10 \text{ in.} \), pitch of the bolt threads \( p = 0.058 \text{ in.} \), modulus of elasticity for steel \( E_s = 30 \times 10^6 \text{ psi} \), modulus of elasticity for the plastic \( E_p = 500 \text{ ksi} \), cross-sectional area of one bolt \( A_s = 0.06 \text{ in.}^2 \), and cross-sectional area of the plastic cylinder \( A_p = 1.5 \text{ in.}^2 \).

Solution 2.5-21  Plastic cylinder and two steel bolts

\( L = 10 \text{ in.} \)

\( p = 0.058 \text{ in.} \)

\( E_s = 30 \times 10^6 \text{ psi} \)

\( A_s = 0.06 \text{ in.}^2 \) (for one bolt)

\( E_p = 500 \text{ ksi} \)

\( A_p = 1.5 \text{ in.}^2 \)

\( n = 1 \) (see Eq. 2-22)

EQUILIBRIUM EQUATION

\[ P_s = \text{tensile force in one steel bolt} \]

\[ P_p = \text{compressive force in plastic cylinder} \]

\[ P_p = 2P_s \quad \text{(Eq. 1)} \]

COMPATIBILITY EQUATION

\[ \delta_s = \text{elongation of steel bolt} \]

\[ \delta_p = \text{shortening of plastic cylinder} \]

\[ \delta_s + \delta_p = np \quad \text{(Eq. 2)} \]

FORCE-DISPLACEMENT RELATIONS

\[ \delta_s = \frac{P_s L}{E_s A_s} \]

\[ \delta_p = \frac{P_p L}{E_p A_p} \quad \text{(Eq. 3, Eq. 4)} \]

SOLUTION OF EQUATIONS

Substitute (3) and (4) into Eq. (2):

\[ \frac{P_s L}{E_s A_s} + \frac{P_p L}{E_p A_p} = np \quad \text{(Eq. 5)} \]